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Propositional Logic Problem Set 1

1. Write a paragraph which says something about Jacques Herbrand and his accomplishments to symbolic logic.
    a. Jacques Herbrand was a French mathematician who was born on February 12th, 1908 and died July 27th, 1931 at the age of only 23. He is known for his work in Mathematical logic and class field theory. His accomplishments include his work in recursion, the development of the Herbrand's theorem, the Herbrand-Ribet theorem, and Herbrand quotient.
2. Write a paragraph which says something about Alfred Horn and his accomplishments to symbolic logic.
    a. Alfred Horn was an American mathematician from New York City who was born on February 17, 1918 and died on April 16, 2001. He is most known for his work in lattice theory and universal algebra, which is the study of algebraic structures. Through his work with universal algebra, in his later life he formed the foundation of logic programming.
3. Write a paragraph which says something about John Allen Robinson and his accomplishments to symbolic logic.
    a. John Allen Robinson was known as a philosopher, mathematician, and a computer scientist. He was also a professor emeritus at Syracuse University. His biggest contribution was the foundation of the automated theorem proving. He also created the unification algorithm which computes a minimal substitution set for a given problem. His unification algorithm also laid the framework for logical programming like Prolog.
4. Suppose:
    a. P :: she needs a doctor
    b. Q :: she needs a lawyer
    c. R :: she has an accident
    d. S :: she is sick
    e. U :: she is injured

- State the following formulas in English:
    - $(S \rightarrow P) \wedge (R \rightarrow Q)$- If she is sick, then she needs a doctor, and, if she has an accident, then she needs a lawyer.
    - $P \rightarrow (S \vee U)$- She is sick or injured if she needs a doctor.
    - $(P \wedge Q) \rightarrow R$- If she needs a doctor or a lawyer, then she had an accident.
    - $(P \wedge Q) \leftrightarrow (S \wedge U)$- She needs a doctor and a lawyer if and only if she is sick and she is injured.
    - $\sim (S \vee U) \rightarrow \sim P$- If she is not sick and she is not injured then she does not need a doctor.

5. Define what is meant by an interpretation for a WFF.

a. Suppose that F is a WFF containing atoms A1, A2, …. , An. An interpretation is an assignment of truth values to A1, A2, …. , An.

6. How many interpretations are there for a WFF with 4 atoms?
    a. 16
7. What does it mean for an interpretation to satisfy a WFF?
    a. An interpretation satisfies a WFF if the formula is true under an interpretation.
8. Write down an interpretation and a WFF with three different atoms for which the interpretation satisfies the WFF.
    a. F = (~P ∨ Q) → R under the interpretation PQR = FTT.
9. What does it mean for an interpretation to falsify a WFF?
    a. An interpretation is said to falsify a WFF if the formula is false under an interpretation.
10. Write down an interpretation and a WFF with two different atoms for which the interpretation falsifies the WFF.
    a. WFF = (~P ∧ Q) under the interpretation PQ = TT.
11. Define what it means for a WFF to be invalid.
    a. A formula is said to be invalid if the formula is not valid.
12. Write down a WFF containing three different atoms which is valid. Also show that it is valid.
    a. (P ∨ Q) → R

P	Q	R	(P ∨ Q)	(R ∨ ~Q)	(P ∨ Q) ∨ (R V ~Q)
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	F	T	T
F	F	F	F	T	T

13. Define what it means for a WFF to be inconsistent.
    a. A WFF is considered inconsistent if it is false under all interpretations.
14. Write down a WFF containing two different atoms which is inconsistent. Also show that it is inconsistent.
    a. (P → Q) ∧ (Q ∧ ~P)

P	Q	(P → Q)	(Q ∧ ~P)	(P → Q) ∧ (Q ∧ ~P)
T	T	T	F	F
T	F	F	T	F

F	T	T	F	F
F	F	T	F	F

15. Define disjunctive normal form.
    a. A formula f is said to be in disjunctive form if F has the form F1 ∨ F2 ∨ … ∨ Fn where each of the Fi is a conjunction of literals.
16. Transform the following into disjunctive normal forms:
    a. (~P ∧ Q) → R

       ~ (~ P ∧ Q) ∨ R (Switcheroo)

       (~ ~ P ∨ ~ Q) ∨ R (De Morgan's)

       (P ∨ ~ Q) ∨ R (Double Negation)

    b. ~ (P ∨ ~ Q) ∧ (S → T)

       ~ (P ∨ ~ Q) ∧ (~ S ∨ T) (Switcheroo)

       (~ P ∨ ~ ~ Q) ∧ (~ S ∨ T) (De Morgan's)

       (~ P ∧ Q) ∧ (~ S ∨ T) (Double Negation)

       ?

    c. (P → Q) → R
       (~ P ∨ Q) → R (Switcheroo)
       ~ (~ P ∨ Q) ∨ R (Switcheroo)
       (~ ~ P ∧ ~ Q) ∨ R (De Morgan's)
       (P ∧ ~ Q) ∨ R (Double Negation)
17. Define conjunctive normal form.
    a. A formula f is said to be in conjunctive form if F has the form F1 ∧ F2 ∧ … ∧ Fn where each of the Fi is a disjunction of literals.
18. Transform the following into conjunctive normal forms:
    a. P ∨ (~ P ∧ Q ∧ R)

       ?

    b. ~ (P → Q)
       ~ (~ P ∨ Q) (Switcheroo)

       (~ ~ P ∧ ~ Q) (De Morgan's)

       (P ∧ ~ Q) (Double Negation)

    c. (P → Q) → R

       (~ P ∨ Q) → R (Switcheroo)

$\sim (\sim P \vee Q) \vee R$ (Switcheroo)

$(\sim \sim P \wedge \sim Q) \vee R$ (De Morgan's)

$(P \wedge \sim Q) \vee R$ (Double Negation)

$R \vee (P \wedge \sim Q)$ (Commutativity)

$((R \vee P) \wedge (R \vee \sim Q)$ (Distributivity)